ELECTRO STATICS

Introduction:

Electrostatics is a branch of physics or electrical engineering which is used to study the electric field and Magnetic field.

field:-

It is a function that specifies the quantity every where in a region (or) space.

Electricfield: The study of interation between the electric charges at rest position is called as Electricfield.

Magneticfield:- The study of interation b/n the electric charges at Moving position is called as Magnetic field.

Due to moving charge current is produced. Due to current carrying conductor it produces the magnetic field. So, the electric field and Magnetic field are interrelated to each other and this field is called as "ElectroMagnetic field".

Applications:-

This 2 fields are used in various applications in design of Emf devices.

1. Electric Machines.
2. Transformers
3. Transmission lines
4. Electric relays. etc..

Uses:-

The laws and principles of Electromagnetic fields are used in design of devices like..

* satellite communication systems
* RADAR, LASER
* Radio/TV

Difference b/n E.C. Theory and F.e Theory:-

C. Theory	field theory
1. It is a powerful tool for analyzing electrical engineering and communication problems. It deals with the only 2 variables i.e., voltage and current.	It is more complex compared to circuit theory because of field theory having no. of variables i.e., Electric field intensity (E), Electric flux density (D), Magnetic field intensity (H), Magnetic flux density (B).
2. It is used in low frequencies i.e., below 1MHz (Mega)	It is used in high frequencies i.e., above 1MHz
3. In this	* In this field theory concept we need another powerful tool for solving the field problems i.e., vector Analysis.

Uses :- Vector Analysis:

Main uses of Vector-Analysis are :

1. To study the e.m.f theory.

2. get the solution's with in a short period of time.

3. save Economy

4. 3-dimensional problems also can be solved.

Vector Algebra:-

Scalar:- It is aquantity which having the only magnitude but no direction.

Ex:- Temperature, pressure.

Vector:- It is aquantity which have both magnitude and direction in a region.

Ex:- velocity, force, Magnetic field intensity, etc..

Graphical representation of vector:-

$O \longrightarrow P$ $\quad |A|, \vec{A}$

$|A|$ is the magnitude, $\vec{A}$ is direction.

Null vector:- A vector whose magnitude is zero is calle[d] Null vector.

Unit vector:- A vector whose magnitude is unity is [called] as unit vector.

- Unit vector is a dimension less vector and serves to specify direction only. It is represented by

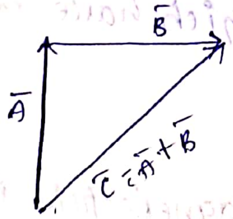
$$\boxed{a_n = \frac{\bar{A}}{|A|}}$$

Scalar and Vector-field:-

Scalar-field:- If the value of quantity at each point in a region is scalar then the field is called as Scalar field.

Vector-field:- If the value of quantity at each point in a region is Vector then the field is called as Vector field.

Addition of Vectors:-



The resultant vector $\bar{c}$ is ~~ap~~ obtained by moving a point along $\bar{A}$ and $\bar{B}$.

Sum of 2 vectors is given by,

$$\boxed{\bar{C} = \bar{A} + \bar{B}}$$

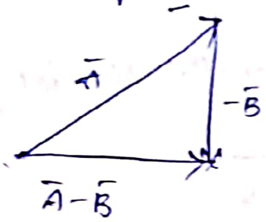
* The addition of 2 vectors obeys commutative law.

$$\boxed{\bar{A} + \bar{B} = \bar{B} + \bar{A}}$$

$$\boxed{(\bar{A} + \bar{B}) + \bar{C} = (\bar{C} + \bar{B}) + \bar{A}}$$

Substraction of Vectors:

To substract $\bar{B}$ from $\bar{A}$ as shown in figure. The substraction of 2 vectors is given by $\bar{A}-\bar{B}$



Multiplication of vectors:-

Multiplication of vectors has been classified into 2 types:

1. Scalar Multiplied by vector (or) vector Multiplied by scalar.
2. Vector Multiplied by another vector.

Vector Multiplied by Scalar:-

when a vector $\bar{A}$ is multiplied by Scalar K multiplies the Magnitude but there is no change in Magnitude direction The resultant vector is indicated by,

$$\boxed{\bar{B} = k|A|}$$

Vector Multiplied by Another Vector:-

when a vector is multiplied by Another vector.

1. Dot product (or) Scalar product
2. Cross product

Dot product:-

The product of 2 vectors $\bar{A}$ and $\bar{B}$ is equal to product of Magnitudes of vectors ($\bar{A}$ and $\bar{B}$) and cosine b/n them.

$$\boxed{\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta}$$

where $|\bar{A}|$ = Magnitude of vector $\bar{A}$

$|\bar{B}|$ = Magnitude of vector $\bar{B}$

θ = Angle b/n 2 vectors $\bar{A}$ and $\bar{B}$

* If 2 vectors are in parallel the dot product of resultant vector is,

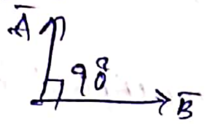
$$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta$$

$$\theta = 0$$

$$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos 0$$

$$\boxed{\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|}$$

– If 2 vectors are in perpendicular then the dot product of resultant vector is,



$$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta$$

$$\theta = 90°$$

$$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos 90°$$

$$\boxed{\bar{A}\cdot\bar{B} = 0}$$

Note:- Consider 2 vectors $\bar{A}$ and $\bar{B}$ i.e.,

$$\bar{A} = A_x\bar{a_x} + A_y\bar{a_y} + A_z\bar{a_z} \longrightarrow (1)$$

$$\bar{B} = B_x\bar{a_x} + B_y\bar{a_y} + B_z\bar{a_z} \longrightarrow (2)$$

where $A_x, A_y, A_z, B_x, B_y, B_z$ are the components of $\bar{A}$ and $\bar{B}$ along their x, y, z axis.

Magnitude of $|\bar{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Magnitude of $\bar{B} = |\bar{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$

Dot product rules:-

* $\bar{a_x} \cdot \bar{a_x} = 1$ $\quad \bar{a_x} \cdot \bar{a_y} = 0$ $\quad \bar{a_y} \cdot \bar{a_z} = 0$

$\bar{a_y} \cdot \bar{a_y} = 1$ $\quad \bar{a_x} \cdot \bar{a_z} = 0$ $\quad \bar{a_z} \cdot \bar{a_x} = 0$

$\bar{a_z} \cdot \bar{a_z} = 1$ $\quad \bar{a_y} \cdot \bar{a_x} = 0$ $\quad \bar{a_z} \cdot \bar{a_y} = 0$

from Dot product rules

eqn (1) & (2) become

$$\bar{A} \cdot \bar{B} = (A_x \bar{a_x} + A_y \bar{a_y} + A_z \bar{a_z}) \cdot (B_x \bar{a_x} + B_y \bar{a_y} + B_z \bar{a_z})$$

$$= A_x B_x^{(1)} + A_y B_y^{(1)} + A_z B_z^{(1)} + A_x B_y^{(0)} + A_x B_z^{(0)} + A_y B_x^{(0)} + A_y B_z^{(0)} + {}^{(0)}A_z B_x + A_z B_y (0)$$

$$\boxed{\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z}$$

~~then~~

* find the magnitude of vectors $\bar{A} = 3\bar{a_x} + 2\bar{a_y} + (-6)\bar{a_z}$,

$\bar{B} = 4\cos\theta \, \bar{a_x} + 4\sin\theta \, \bar{a_y} + 5\bar{a_z}$

Soln: Given that $\bar{A} = 3\bar{a_x} + 2\bar{a_y} + (-6)\bar{a_z}$

$$|\bar{A}| = \sqrt{3^2 + 2^2 + (-6)^2}$$

$$= \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

$$|\bar{B}| = \sqrt{(4\cos\theta)^2 + (4\sin\theta)^2 + 5^2}$$

$$= \sqrt{16 + 25} = \sqrt{41}$$

* use Dot product to find angle between 2 vectors

i) $3\bar{a}_x - 2\bar{a}_y + \bar{a}_z$, $-\bar{a}_x + 2\bar{a}_y + 7\bar{a}_z$

ii) $2\bar{a}_x$ & $4\bar{a}_x - 3\bar{a}_y + 5\bar{a}_z$

i) sol:

let $\bar{A} = 3\bar{a}_x - 2\bar{a}_y + \bar{a}_z$, $\bar{B} = -\bar{a}_x + 2\bar{a}_y + 7\bar{a}_z$

$(\bar{A}\cdot\bar{B}) = (3\bar{a}_x - 2\bar{a}_y + \bar{a}_z)\cdot(-\bar{a}_x + 2\bar{a}_y + 7\bar{a}_z)$

$= -3 - 4 + 7 = 0$

$|A| = \sqrt{(3)^2 + (2)^2 + 1^2} = \sqrt{14}$

$|B| = \sqrt{1^2 + 2^2 + 7^2} = \sqrt{54}$

$\bar{A}\cdot\bar{B} = |A||B|\cos\theta$

$\therefore$ $0 = \sqrt{14}\sqrt{54}\cos\theta$

$\boxed{\theta = 90^\circ}$

ii) sol:

let $\bar{A} = 2\bar{a}_x$, $\bar{B} = 4\bar{a}_x - 3\bar{a}_y + 5\bar{a}_z$

$(\bar{A}\cdot\bar{B}) = (2\bar{a}_x)\cdot(4\bar{a}_x - 3\bar{a}_y + 5\bar{a}_z)$

$= 8$

$|A| = \sqrt{4} = 2$

$|B| = \sqrt{16+9+25} = \sqrt{50}$

$\cos\theta = \dfrac{(\bar{A}\cdot\bar{B})}{|A||B|} = \dfrac{8}{2\sqrt{50}} = \dfrac{4}{\sqrt{50}} = 0.5656$

$= 55.55$

* i) $4\bar{a_x} + 3\bar{a_y} + 2\bar{a_z}$ and $-5\bar{a_x} + 3\bar{a_y} + 6\bar{a_z}$

ii) $5\bar{a_x} + 6\bar{a_y}$. and $5\bar{a_x} - 5\bar{a_y} + \cancel{8}\,\bar{a_z}$

i sol: let $\bar{A} = 4\bar{a_x} + 3\bar{a_y} + 2\bar{a_z}$, $\bar{B} = -5\bar{a_x} + 3\bar{a_y} + 6\bar{a_z}$

$$\bar{A}\cdot\bar{B} = (4\bar{a_x} + 3\bar{a_y} + 2\bar{a_z})\cdot(-5\bar{a_x} + 3\bar{a_y} + 6\bar{a_z})$$

$$= -20 + 9 + 12 = 1$$

$$|\bar{A}| = \sqrt{16+9+4} = \sqrt{29}$$

$$|\bar{B}| = \sqrt{25+9+36} = \sqrt{70}$$

$$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta$$

$$1 = \sqrt{29}\sqrt{70}\cos\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{29}\sqrt{70}} = \cancel{100895}$$

$$= 0.022$$

$$\theta = 88.728$$

ii) sol:

let $\bar{A} = 5\bar{a_x} + 6\bar{a_y}$, $\bar{B} = 5\bar{a_x} - 5\bar{a_y} + \bar{a_z}$

$$(\bar{A}\cdot\bar{B}) = (5\bar{a_x} + 6\bar{a_y})\cdot(5\bar{a_x} - 5\bar{a_y} + \bar{a_z}) = 25 - 30 = -5$$

$$|\bar{A}| = \sqrt{25+36} = \sqrt{61}, \quad |\bar{B}| = \sqrt{25+25+1} = \sqrt{51}$$

$$(\bar{A}\cdot\bar{B}) = |\bar{A}||\bar{B}|\cos\theta$$

$$\cos\theta = \frac{-5}{\sqrt{61}\sqrt{51}} = -0.089$$

$$\theta = 84.856$$

* cross product:-

consider 2 vectors $\bar{A}$ and $\bar{B}$. the crossproduct of 2 vectors is equal to the product of magnitudes of these vectors and sign of angle b/n them.

Mathematically $\boxed{\bar{A}\times\bar{B} = |\bar{A}||\bar{B}| \sin\theta \, \hat{a_n}}$

where $|\bar{A}|$ = Magnitude of vector $\bar{A}$

$|\bar{B}|$ = Magnitude of vector $\bar{B}$

θ = Angle b/n 2 vector

$\hat{a_n}$ = unit vector.

$\therefore$ unit vector $\hat{a_n} = \dfrac{\bar{A}\times\bar{B}}{|\bar{A}||\bar{B}| \sin\theta}$

[If $\theta = 0$]

- If 2 vectors are in parallel then crossproduct is,

$\theta = 0$; $\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|.\sin 0$

$$\boxed{\bar{A}\times\bar{B} = 0}$$

- If 2 vectors are perpendicular then crossproduct is

$\theta = 90°$

$\bar{A}\times\bar{B} = |\bar{A}||\bar{B}| \sin 90°$

$$\boxed{\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|}$$

cross product of 2 vectors are represented in matrix-form, then:

$$\bar{A}\times\bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

cross-product rules:-

$\bar{a_x}\times\bar{a_x} = 0$ $\quad$ $\bar{a_x}\times\bar{a_y} = \bar{a_z}$ $\quad$ $\bar{a_y}\times\bar{a_z} = \bar{a_x}$

$\bar{a_y}\times\bar{a_y} = 0$ $\quad$ $\bar{a_x}\times\bar{a_z} = \bar{a_x}$ $\quad$ $\bar{a_z}\times\bar{a_x} = \bar{a_y}$

$\bar{a_z}\times\bar{a_z} = 0$ $\quad$ $\bar{a_y}\times\bar{a_x} = \bar{a_z}$ $\quad$ $\bar{a_z}\times\bar{a_y} = \bar{a_x}$

consider 2 vectors $\bar{A}$ and $\bar{B}$ is

$\bar{A} = A_x\bar{a_x} + A_y\bar{a_y} + A_z\bar{a_z}$

$\bar{B} = B_x\bar{a_x} + B_y\bar{a_y} + B_z\bar{a_z}$

then

$\bar{A}\times\bar{B}$ is $a_x(A_yB_z - A_zB_y) - a_y(A_xB_z - A_zB_x) + a_z(A_xB_y - B_xA_y)$

* Given vectors $\bar{A} = 3\bar{a_x} + 4\bar{a_y} + \bar{a_z}$; $\bar{B} = 2\bar{a_y} - 5\bar{a_z}$. find the angle b/n $\bar{A}$ and $\bar{B}$ vectors using dot product and crossproduct.

Sol: Given $\bar{A} = 3\bar{a_x} + 4\bar{a_y} + \bar{a_z}$; $\bar{B} = 2\bar{a_y} - 5\bar{a_z}$ $\quad$ find unitvector

$(\bar{A}\cdot\bar{B}) = (3\bar{a_x} + 4\bar{a_y} + \bar{a_z})\cdot(2\bar{a_y} - 5\bar{a_z})$

$= 8 - 5 = 3$

$|\bar{A}| = \sqrt{9+16+1} = \sqrt{26}$

$|\bar{B}| = \sqrt{4+25} = \sqrt{29}$

Dot product:

$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta$

$\cos\theta = \dfrac{3}{\sqrt{26}\sqrt{29}} = 0.109$

$\theta = 83.727$

$$\bar{A}\times\bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ 3 & 4 & 1 \\ 0 & 2 & -5 \end{vmatrix}$$

$$= a_x(-20-2) - a_y(-15+0) + a_z(6-0)$$

$$= -22a_x + 15a_y + 6a_z$$

$$\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|\sin\theta$$

$$\sin\theta = \frac{|-22\bar{a}_x + 15\bar{a}_y + 6\bar{a}_z|}{\sqrt{26}\sqrt{29}}$$

$$|\bar{A}\times\bar{B}| = \sqrt{484+225+36} = \sqrt{745}$$

$$\sin\theta = \frac{\sqrt{745}}{\sqrt{26}\sqrt{29}} = 0.994$$

$$\theta = 83.72^0$$

unit vector $a_n = \dfrac{(\bar{A}\times\bar{B})}{|\bar{A}||\bar{B}|\sin\theta} = \dfrac{-22\bar{a}_x + 15\bar{a}_y + 6\bar{a}_z}{\sqrt{26}\sqrt{29}(0.994)}$

$$= \frac{-22\bar{a}_x + 15\bar{a}_y + 6\bar{a}_z}{27.294}$$

$$= -0.806\bar{a}_x + 0.549\bar{a}_y + 0.219\bar{a}_z$$

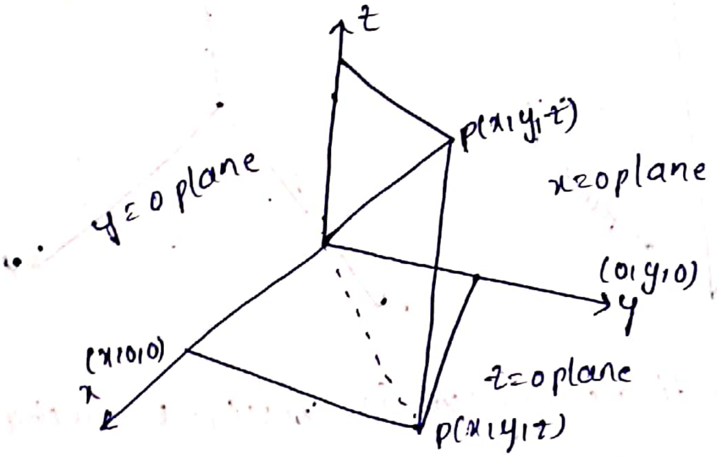
co-ordinate system (or) orthogonal system:-

co-ordinate system is a powerful tool which is need to solve the problems in a field theory concept. this system is also called as orthogonal system. The system in which co-ordinates are mutually perpendicular. so the orthogonal system is divided into 3types i) Rectangular (or) cartesian system

ii) polar (or) cylindrical co-ordinate system

iii) spherical co-ordinate system.

1. Rectangular co-ordinate system:-



It is formed by 3 mutual perpendicular straight lines and is called as Rectangular co-ordinate system. These 3 straight lines are called x,y,z co-ordinates. The range of this co-ordinates is

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$

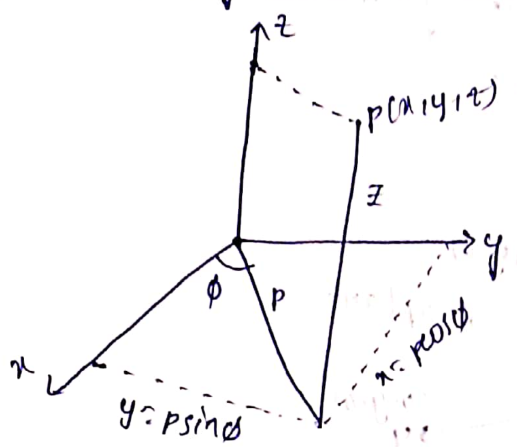
x,y,z lies '+ve' and '-ve' sides of planes. Here the interaction point of these axis is called region (or) space. To represent the component of any vector along this axis we use unit vectors, i.e., $\bar{a}_x, \bar{a}_y, \bar{a}_z$. Any point $p(x,y,z)$ in in region from the figure we can find out the point 'p' (or) vector. In this Rectangular co-ordinate system x,y,z are the components of a given vector and unit vectors are $\bar{a_x}, \bar{a_y}, \bar{a_z}$. then we can find out of the differential length ($\bar{dl}$), differential surface($\bar{ds}$), differential volume ($\bar{dv}$).

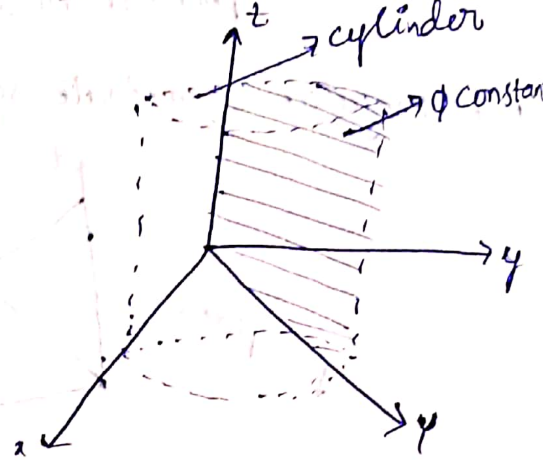
$$\bar{dl} = dx\bar{a_x} + dy\bar{a_y} + dz\bar{a_z}$$

$$\bar{ds} = \underset{x=0}{dydz\,\bar{a_x}} + \underset{y=0}{dxdz\,\bar{a_y}} + \underset{z=0}{dxdy\,\bar{a_z}}$$

$$\bar{dv} = dxdydz$$

* **Polar (or) cylindrical co-ordinate system:-**





The polar co-ordinate system is convineat method when ever dealing with the symmetrical problem. In this system the components of a given vector is represented by r, ϕ, z. The range of this co-ordinates are $0 \leq r \leq \infty$

r = radius $\qquad 0 \leq \phi \leq 2\pi$

ϕ = position $\qquad -\infty \leq z \leq \infty$

z = length

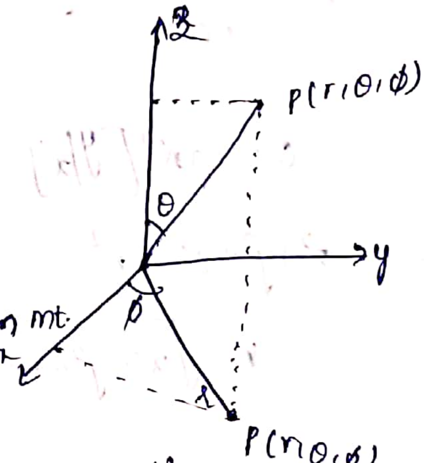
Here the three co-ordinates r, ϕ, z and unit vectors are $\bar{a_r}, \bar{a_\phi}, \bar{a_z}$, then we find out the differential length, differential surface($\bar{ds}$), differential volume ($\bar{dv}$).

$d\bar{l} = dr\,\bar{a_x} + r\,d\phi\,\bar{a_y} + dz\,\bar{a_z}$

$d\bar{s} = r\,d\phi\,dz\,\bar{a_r} + dr\,dz\,\bar{a_\phi} + r\,dr\,d\phi\,\bar{a_z}$

$d\bar{v} = r\,dr\,d\phi\,dz$

* Spherical co-ordinate system:-



In spherical co-ordinate system, the spherical co-ordinates are r, θ, ϕ

where r = radius from origin to P in mt.

θ = angle measured from reference axis z in degrees or radians

ϕ = angle measured from reference axis x

The range of these co-ordinates are

$$0 \le r \le \infty$$

$$0 \le \theta \le 2\pi$$

$$0 \le \phi \le 2\pi$$

The spherical components are $d\hat{r}$, $d\theta$, $d\phi$ and unit vectors are $\bar{a_r}, \bar{a_\theta}, \bar{a_\phi}$.

then $d\bar{l} = dr\,\bar{a_r} + r\,d\theta\,\bar{a_\theta} + r\sin\theta\,d\phi\,\bar{a_\phi}$

$d\bar{s} = r\,d\theta\, r\sin\theta\,d\phi\,\bar{a_r} + dr\, r\sin\theta\,d\phi\,\bar{a_\theta} + dr\, r\,d\theta\,\bar{a_\phi}$

$r=0 \qquad \theta=0 \qquad \phi=0$

$= r^2\sin\theta\,d\theta\,d\phi\,\bar{a_r} + r\sin\theta\,dr\,d\phi\,\bar{a_\theta} + r\,dr\,d\theta\,\bar{a_\phi}$

$d\bar{v} = r^2\sin\theta\,dr\,d\theta\,d\phi$

*Relation's between Rectangular to Polar coordinate system:

We know that the co-ordinates are x, y, z. (In RCS)

In P.C.S the co-ordinates are r, ϕ, z

$\therefore$ $x = r\cos\theta$, $y = r\sin\theta$, $z = z$

- $\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} \Rightarrow \frac{y}{x} = \tan\theta$

$\theta = \tan^{-1}[y/x]$

- $x^2+y^2+z^2 = r^2$

$r = \sqrt{x^2+y^2+z^2}$

* Relation's b/n RCS to SCS:—

w.k.t. In RCS the co-ordinates are x, y, z.

In SCS the co-ordinates are r, θ, ϕ

$\therefore x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

- $\cos\theta = z/r$

$\theta = \cos^{-1}[z/r]$

- $y/x = \frac{\cancel{r}\cancel{\sin\theta}\sin\phi}{\cancel{r}\cancel{\sin\theta}\cos\phi} \Rightarrow \tan\phi = y/x$

$\phi = \tan^{-1}[y/x]$

- $r^2 = x^2+y^2+z^2$

$r = \sqrt{x^2+y^2+z^2}$

* Operator :- (∇)

Dell is a vector operator (or) gradient. It is indicated by ∇.

the operator ∇ is

$$\nabla = \frac{\partial}{\partial x}\bar{a_x} + \frac{\partial}{\partial y}\bar{a_y} + \frac{\partial}{\partial z}\bar{a_z}$$

there are 3 ways, the operator ∇ can act

1. on Scalar function.
2. Dot product
3. Cross product

* on scalar function:-

ϕ is a Scalar function then,

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a_x} + \frac{\partial\phi}{\partial y}\bar{a_y} + \frac{\partial\phi}{\partial z}\bar{a_z}$$

* for dot product the vector equation is $\bar{V} = V_x\bar{a_x} + V_y\bar{a_y} + V_z\bar{a_z}$.

then $$\nabla\cdot\bar{V} = \left[\frac{\partial}{\partial x}\bar{a_x} + \frac{\partial}{\partial y}\bar{a_y} + \frac{\partial}{\partial z}\bar{a_z}\right]\cdot\left[V_x\bar{a_x} + V_y\bar{a_y} + V_z\bar{a_z}\right]$$

$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

* for cross product

$$\nabla\times\bar{V} = \begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right]\bar{a_x} - \left[\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right]\bar{a_y}$$

$$+ \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]\bar{a_z}$$

$$\boxed{\nabla\times\bar{V} = 0}$$

* **columb's law / Inverse Square law :-** $Q_1 \bullet \xleftarrow{\quad d \quad} \xrightarrow{\quad} \bullet Q_2$

It states that the force of attraction or repulsion b/n the 2 point charges is directly proportional to the product of magnitide of 2 point charges and inversely proportional to the square of the distance b/n them.

from figure,

let Q_1 and Q_2 be 2 point charges. 'd' denotes distance b/n 2 charges. so from the statement,

$$F \propto Q_1 Q_2$$

$$F \propto \frac{1}{d^2}$$

$$F \propto \frac{Q_1 Q_2}{d^2}$$

$$F = \frac{K Q_1 Q_2}{d^2} \qquad K = \frac{1}{4\pi\varepsilon}$$

$$\boxed{F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2}}$$

from the equation F = force in Newtons

Q_1 and Q_2 are 2 point charges in columbs

d is distance b/n 2 charges in meters

ε permitivity of media.

$\varepsilon = \varepsilon_0 \varepsilon_r$

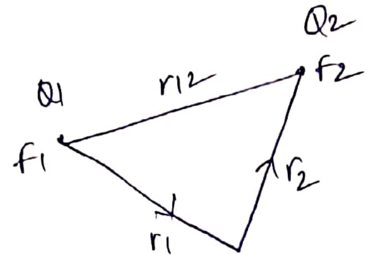
ε_0 is absolute permitivity $= 8.85 \times 10^{-12}$

ε_r is relative permitivity

$\varepsilon_r = 1$ for air $\qquad \varepsilon_r = 5$ to 10 for glass

$\varepsilon_r = 2-3$ for paper

Vector form of columbs law:-



- Q2 feel a force from the presence of charge Q_1. f1 and f2 forces are equal but opposite in direction.

from the figure, Q_1 = first charge in cd

Q_2 = Second charge in colu

r_1 = location of charge Q_1

r_2 = Location of charge Q_2

r_{12} = distance vector along line joining of Q_1 and Q_2 charges

$$\therefore \quad r_{12} = \cancel{r_{21}}\ r_2 - r_1$$

$$|r_{12}| = |r_2 - r_1|$$

the direction of r_{12} is a_{12}

$$\boxed{a_{12} = \frac{r_{12}}{|r_{12}|}}$$

$$r_{21} = r_1 - r_2 \quad \text{or} \quad -(r_2 - r_1)$$

$$\boxed{r_{21} = -r_{12}}$$

The force on Q_2 due to Q_1 is ac

according to columbs law

$$\boxed{f_2 = \frac{Q_1 Q_2}{4\pi\varepsilon r_{12}^2} \bar{a}_{12}}$$

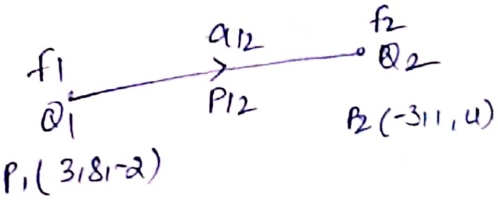
force on Q_1 due to charge Q_2 is,

$$\boxed{f_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{21}^2} \vec{a}_{21}}$$

So, finally $\boxed{f_2 = -f_1}$ or $\boxed{f_1 = -f_2}$

f_1 and f_2 ~~forces~~ forces are equal but opposite in direction.

* find the potential and Electric field due to any type of charge distribution. a charge $Q_2 = 10$ micro colum is located in an air at P_2 of $(-3,1,4)$ meters. find the force on Q_2 due to $Q_1 = 33\mu C$ located at P_1 $(3,8,-2)$m.



Sol:- $Q_2 = 10\mu C = 10\times10^{-6}C$

P_2 $(-3,1,4)$m

$f_2 = ?$

$Q_1 = 33\mu C = 33\times10^{-6}$

$P_1 = (3,8,-2)^m$

$P_1 = 3\bar{a}_x + 8\bar{a}_y - 2\bar{a}_z$

$P_2 = -3\bar{a}_x + \bar{a}_y + 4\bar{a}_z$

$\varepsilon = \varepsilon_0\varepsilon_r$

$= 8.85\times10^{-12} \times 1$

$\varepsilon = 8.85\times10^{-12}$

$$P_{12} = P_2 - P_1$$

$$= (-3\bar{a_x} + \bar{a_y} + 4\bar{a_z}) - (+3\bar{a_x} + 8\bar{a_y} - 2\bar{a_z})$$

$$P_{12} = -6\bar{a_x} - 7\bar{a_y} + 6\bar{a_z}$$

$$|P_{12}| = \sqrt{36+36+49}$$

$$= \sqrt{121} = 11$$

$$a_{12} = \frac{P_{12}}{|P_{12}|} = \frac{-6\bar{a_x} - 7\bar{a_y} + 6\bar{a_z}}{11}$$

$$= -0.5454\,\bar{a_x} - 0.6363\,\bar{a_y} + 0.5454\,\bar{a_z}$$

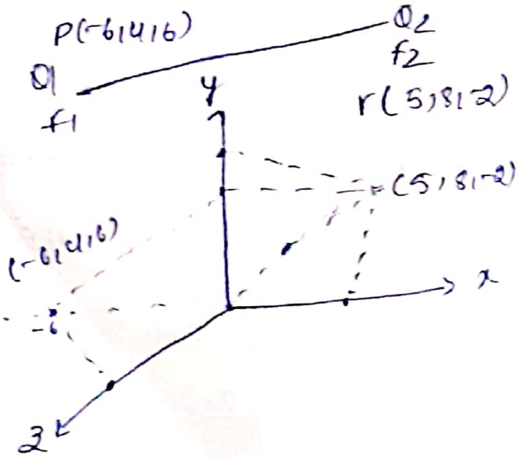
$$F_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot P_{12}^2} \times \bar{a_{12}}$$

$$= \frac{33\times10^{-6} \times 10\times10^{-6}}{4\pi \times 8.85\times10^{-12} \times 11^2}[-0.545\bar{a_x} - 0.636\bar{a_y} + 0.545\bar{a_z})$$

$$= 0.0245\,[-0.545\bar{a_x} - 0.636\bar{a_y} + 0.545\bar{a_z}]$$

$$= (-0.01336\bar{a_x} - 0.0155\bar{a_y} + 0.01336\bar{a_z})\ N$$

* A charge $Q_1 = -20\mu C$ is located at point $p(-6, 4, 6)$ and charge $Q_2 = 50\mu C$ is located at point $r(5, 8, -2)$ in a free space. find the force on Q_2 due to Q_1 in vector form the distance given in m.



Sol:-

$Q_1 = -20\mu C = -20\times10^{-6}$

$Q_2 = 50\mu C = 50\times10^{-6}$

$P = -6\bar{a_x} + 4\bar{a_y} + 6\bar{a_z}$

$r = 5\bar{a_x} + 8\bar{a_y} - 2\bar{a_z}$

$\varepsilon = 8.85\times10^{-12}$

$$P_r = \bar{r} - \bar{p} = (5\hat{a}_x + 2\hat{a}_y - 20\hat{a}_z) - (-6\hat{a}_x + 4\hat{a}_y + 10\hat{a}_z)$$

$$= 11\hat{a}_x + 4\hat{a}_y + 8\hat{a}_z$$

$$|P_r| = \sqrt{11^2 + 4^2 + 8^2} \qquad \hat{a}_{pr} = \frac{11\hat{a}_x + 4\hat{a}_y + 8\hat{a}_z}{14.17}$$

$$= 14.17 \qquad\qquad = 0.7760\hat{a}_x + 0.282\hat{a}_y + 0.564\hat{a}_z$$

$$F_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 P_r^2}\,\hat{a}_{pr}$$

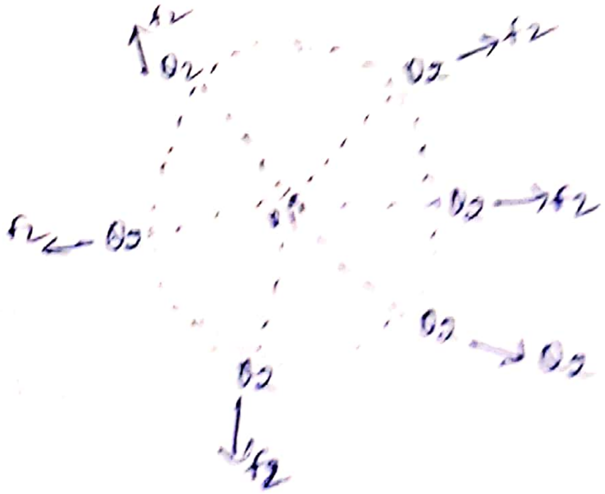
$$= \frac{-20\times10^{-6}\times 50\times10^{-6}}{4\pi\times 8.85\times10^{-12}\times(14.17)^2}\,(0.7760\hat{a}_x + 0.2820\hat{a}_y + 0.5640\hat{a}_z)$$

$$= \frac{-1000}{22{,}330{,}211}\times(0.7760\hat{a}_x + 0.2820\hat{a}_y + 0.5640\hat{a}_z)$$

$$= -0.0447\,(0.7760\hat{a}_x + 0.2820\hat{a}_y + 0.5640\hat{a}_z)$$

$$F_1 = -0.0347\hat{a}_x - 0.01260\hat{a}_y + 0.02520\hat{a}_z \text{ N}$$

Electric field Intensity / Electric field strength (E):-



Consider a point charge Q_1 as shown in figure. If any other charge Q_2 brought nearer to Q_1, it experiences force F_2 on Q_2 due to Q_1 as shown in fig.

Thus there exists a region around the charge which is called Electric field.

According to columbs law,

force on Q_2, due to Q_1 is

$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon r_{12}^2} \bar{a_r}$$

Defenition :-

The force per unit charge in called as electric field intensity. It is indicated by $\bar{E}$ and measured in Newton/colum.

Mathematically, $\bar{E} = \frac{F_2}{Q_2}$

$$= \frac{Q_1 \cancel{Q_2}}{4\pi\varepsilon_0 r_{12}^2} \frac{\bar{a_r}}{\cancel{Q_2}}$$

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \bar{a_r}}$$

Note:- If there are n point charges the field intensity at a point is equal to vector sum of field due to n charges.

$$\boxed{E_n = E_1 + E_2 + E_3 + \cdots + E_n}$$

Salient features of $\bar{E}$:-

⇒ It is a vector ~~both~~ having both magnitude and direction.

⇒ measured in N/c (or) V^{olt}/mt

⇒ It depends upon media

⇒ It depends upon location of charge

⇒ It depends upon distance ~~of~~ b/n charges. ~~and~~ as

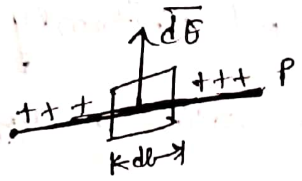
Electric field Intensity due to continuous charge distribution:-

In Electro static field the electric-field intensity deals with point charge and different types of continuous charges.

1. point charge +Q
2. Line charge
3. surface charge
4. volume charge

Line charge:-

Here the charge is distributed uniformly and continuously through out the line is called line charge as shown in fig.:



where 'ρl' is line charge density and is defined as charge per unit length.

i.e., $\boxed{\rho_L = Q/L}$ C/m

- A small change in charge and small elementary length then, $\rho_L = \frac{dQ}{dL}$

$$dQ = \rho_L dL$$

$$Q = \int_l \rho_L dL$$

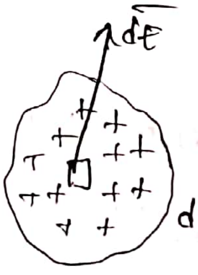
with reference to line charge density ρ_L the differential charge produces differential electric field intensity

$$\therefore \bar{dE} = \frac{dQ}{4\pi\epsilon r^2} . \bar{a_r}$$

$$\bar{dE} = \frac{\rho_L . dL}{4\pi\epsilon r^2} . \bar{a_r}$$

$$\boxed{\bar{E} = \int_l \frac{\rho_L . dL}{4\pi\epsilon r^2} \bar{a_r}}$$

Surface charge:-



$dQ = \rho_s ds$

Here the charge is distributed uniformly and continuous through out the surface is called as surface charge.

where ρ_s is the surface charge density and is defined as charge per unit surface

$$\boxed{\rho_s = \frac{Q}{S} \; C/m^2}$$

- A small charge in charge and small elementary surface.

then, $\rho_s = \frac{dQ}{ds}$

$$dQ = \rho_s \, ds$$

$$Q = \int_s \rho_s \, ds$$

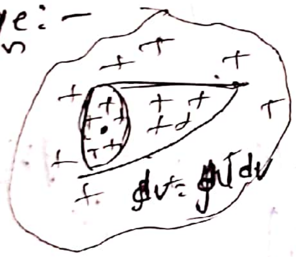
with reference to surface charge density ρ_s is the differential charge produces differential electric field Intensity

$$d\bar{E} = \frac{dQ}{4\pi\varepsilon r^2} \bar{a_r}$$

$$d\bar{E} = \frac{\rho_s ds}{4\pi\varepsilon r^2} \bar{a_r}$$

$$\boxed{\bar{E} = \int_s \frac{\rho_s ds}{4\pi\varepsilon r^2} \bar{a_r}}$$

volume charge:-



Here the charge is distributed uniformly and continuous through out the volume is called as volume charge.

where ρ_v is the ~~surface~~ volume charge density and is defined as charge per unit volume.

$$\boxed{\rho_v = \frac{Q}{V} \ C/m^3}$$

- A small change in charge and small elementary volume.

then, $\rho_v = \frac{dQ}{dv}$

$dQ = \rho_v dv$

$Q = \int_v \rho_v dv$

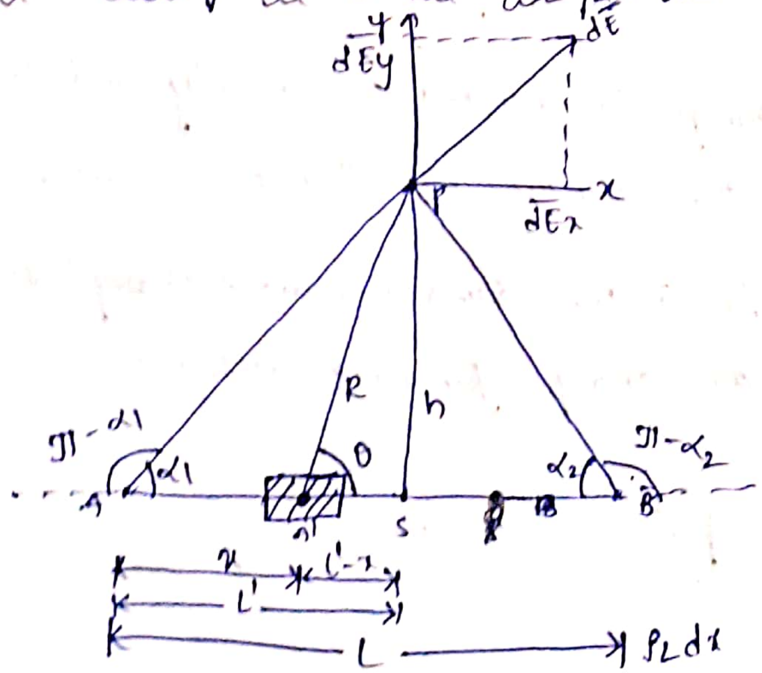
with reference to volume charge density ρ_v the differential charge produces differential electric field Intensity.

$$d\bar{E} = \frac{dQ}{4\pi\varepsilon r^2}\bar{a_r}$$

$$d\bar{E} = \frac{\rho_v dv}{4\pi\varepsilon r^2}\bar{a_r}$$

$$\boxed{\bar{E} = \int_v \frac{\rho_v dv}{4\pi\varepsilon r^2}\bar{a_r}}$$

Electric field intensity due to line charge / wire :-



consider a length of wire AB of L mt which has uniform charge continuously distributed through out the line. Its line charge density is 'ρ_L'. let us determine electric field intensity with a line charge AB. from the figure the point 'p' is a distance of 'h mts' from 's' which is at a distance of 'l' from end of 'A'.

consider an differential element 'dx', which is at a distance of 'x mt. from 'A'. ~~we kn~~

We know that,

$$\text{Line charge density } (\rho_L) = \frac{Q}{L}$$

$$\rho_L = \frac{dQ}{dL} \quad [\because dL = dx]$$

$$\rho_L = \frac{dQ}{dx}$$

$$\boxed{dQ = \rho_L dx} \longrightarrow (1)$$

with reference of line charge density the differential electric field intensity is,

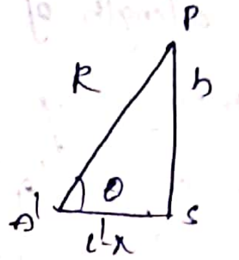
$$\bar{E} = \frac{Q}{4\pi\varepsilon R^2}\bar{a}_r$$

$$d\bar{E} = \frac{dQ}{4\pi\varepsilon R^2}\bar{a}_r$$

$$d\bar{E} = \frac{\rho_L dx}{4\pi\varepsilon R^2}\bar{a}_r \longrightarrow (2)$$

~~from~~ the

from the figure,



$\cos\theta = \frac{l'-x}{R}$, $\sin\theta = h/R$

$\tan\theta = \frac{h}{l'-x}$

$h = R\sin\theta$

$R = \frac{h}{\sin\theta}$

$h = (l'-x)\tan\theta$

$\boxed{R = h\operatorname{cosec}\theta} \longrightarrow (a)$

$l'-x = \frac{h}{\tan\theta}$

$l'-x = h\cot\theta$

diff. w.r.t. θ

$\frac{dl'}{d\theta} - \frac{dx}{d\theta} = h\frac{d}{d\theta}\cot\theta$

$\cancel{-}\frac{dx}{d\theta} = h(\cancel{-}\operatorname{cosec}^2\theta)$

$\boxed{dx = h\operatorname{cosec}^2\theta \, d\theta} \longrightarrow (b)$

substitute eqn (a) and (b) in eqn (2)

$d\bar{E} = \frac{\rho_L \cancel{h}\cancel{\operatorname{cosec}^2}\theta\, d\theta}{4\pi\varepsilon h^{\cancel{2}}\cancel{\operatorname{cosec}^2}\theta}\bar{a_r}$

$\boxed{d\bar{E} = \frac{\rho_L d\theta}{4\pi\varepsilon h}\bar{a_r}} \longrightarrow (3)$

let $d\bar{E}$ is resultant vector from dE_x and dE_y.

from the figure $d\bar{E} = d\bar{E}_x + d\bar{E}_y$

where $d\bar{E}_x = d\bar{E}\cos\theta$

$d\bar{E}_y = d\bar{E}\sin\theta$

case-1:-

$$d\bar{E}_x = d\bar{E}\cos\theta$$

$$\bar{E}_x = \int_{\alpha_1}^{\pi-\alpha_2} d\bar{E}\cos\theta\, d\theta$$

$$= d\bar{E}\left[\sin\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$= d\bar{E}\left[\sin(\pi-\alpha_2) - \sin\alpha_1\right]$$

$$= d\bar{E}\left[\sin\alpha_2 - \sin\alpha_1\right]$$

case-2:-

$$d\bar{E}_y = d\bar{E}\sin\theta$$

$$\bar{E}_y = \int_{\alpha_1}^{\pi-\alpha_2} d\bar{E}\sin\theta\, d\theta$$

$$= d\bar{E}\left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$= d\bar{E}\left[-\left[\cos(\pi-\alpha_2) + \cos\alpha_1\right]\right]$$

$$= d\bar{E}\left[\cos\alpha_2 + \cos\alpha_1\right]$$

take $\alpha_1 = 0$, $\alpha_2 = 0$

$\bar{E}_x = 0$ $\quad \bar{E}_y = 2d\bar{E}$

$\bar{E} = \bar{E_x} + \bar{E_y}$

$\bar{E} = 0+2d\bar{E}$

$\bar{E} = 2d\bar{E}$

$\bar{E} = 2\left[\frac{\rho_l \cdot d\theta}{4\pi\varepsilon h} \cdot \bar{a_r}\right]$

$$\boxed{\bar{E} = \frac{\rho_l \cdot d\theta}{2\pi\varepsilon h}\bar{a_r}}$$

* A charge is distributed on x-axis of cartesian system having a line charge density of $3x^2 \mu C/m$. find the total charge over the length of 10m.

Sol^n:-

$\rho_L = Q/L$

$\rho_L = dQ/dL$

$dQ = dL\,\rho_L$

$Q = \int_0^{10} dL\,\rho_L$

$= \int_0^{10} 3x^2\,dx \times 10^{-6}$

$= \frac{3x^3}{3} = [x^3]_0^{10} \times 10^{-6}$

$= 10^{-3}$ columbs

Q. find the total charge inside a volume having volume charge density as $10z^2 e^{-0.1x} \sin\pi y$. The volume is defined b/n $-2 \le x \le 2$, $0 \le y \le 1$, $3 \le z \le 4$

Sol:

$\rho_v = 10z^2 e^{-0.1x} \sin\pi y$

$$\rho_v = \frac{Q}{v} = \frac{dQ}{dv}$$

$$dQ = \rho_v dv$$

$$Q = \int \rho_v \cdot dv$$

$$= \int_3^4 \int_0^1 \int_{-2}^{2} 10z^2 e^{-0.1x} \sin\pi y \, dx\,dy\,dz$$

$$= \int_3^4 \int_0^1 10z^2 \sin\pi y \left[\frac{e^{-0.1x}}{-0.1}\right]_{-2}^{2} dy\,dz$$

$$= \int_3^4 \int_0^1 10z^2 \sin\pi y \left[\frac{e^{-0.1\times 2}}{-0.1} - \frac{e^{-0.1\times -2}}{-0.1}\right] dy\,dz$$

$$\int_3^4 \int_0^1 10z^2 \sin\pi y \left[-8.187 + 12.214\right] dy\,dz$$

$$= \int_3^4 \int_0^1 10z^2 \sin\pi y \left[4.027\right] dy\,dz$$

$$= 4.027 \int_3^4 10z^2 \left[\frac{-\cos\pi y}{\pi}\right]_0^1 dz$$

$$= 4.027 \int_3^4 10z^2 \left[-\frac{(-1)}{\pi} + \frac{1}{\pi}\right]$$

$=$

$$= 2.5636 \int_3^4 10z^2\, dz$$

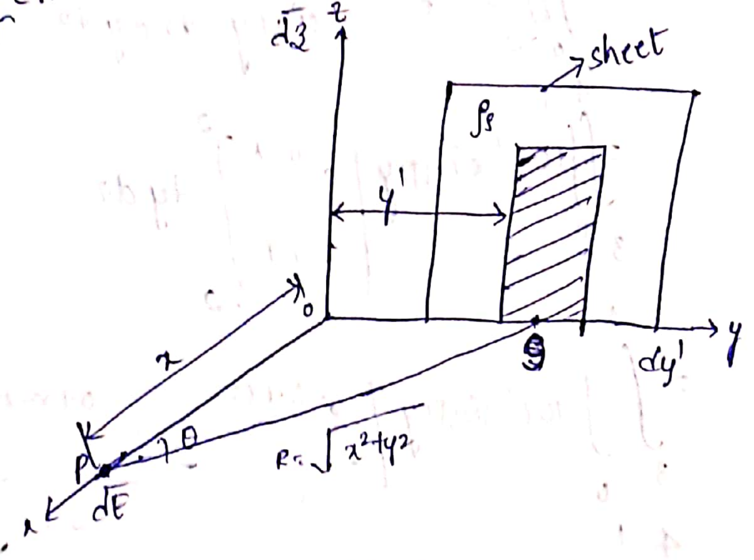
$$= 2.5636 \; 10\left[\frac{z^3}{3}\right]_3^4$$

$$= 2.5636 \times 10 \times \left[\frac{4^3}{3} - \frac{3^3}{3}\right]$$

$$= 25.636\left[21.333 - 9\right]$$

$= 316.168$ columbs.

* Electric field intensity due to surface charge/sheet of charge:-



- consider an infinite sheet of charge with a uniform surface charge density 'ρ_s' as shown in figure.
- let us obtain electric field intensity due to sheet of charge placed in yz-plane at a distance of 'y'' from z-axis and differential width 'dy''.
- where the point 'P' is on x-axis. let R is the distance of 'P' from the sheet of charge to 'P'.

from the figure

$$\cos\theta = \frac{x}{R} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\sin\theta = \frac{y'}{R} = \frac{y'}{\sqrt{x^2+y^2}}$$

$$\tan\theta = \frac{y'}{x}$$

The surface charge density is defined as,

$$\rho_s = Q/s$$

$$\rho_s = dQ/ds$$

$$\boxed{\therefore ds = dL\,dy'}$$

$$\rho_s = \frac{dQ}{dL\,dy'}$$

$$\rho_s = \frac{\rho_L}{dy'}$$

$$\boxed{\rho_L = \rho_s dy'} \longrightarrow (1)$$

w.r.t. electric field intensity at a point p due to line charge is,

$$d\bar{E} = \frac{\rho_L}{2\pi\varepsilon h}\bar{a_r} \longrightarrow (2)$$

substitute (1) in (2)

$$d\bar{E} = \frac{\rho_s dy'}{2\pi\varepsilon R} \qquad \boxed{h = R}$$

$$d\bar{E} = \frac{\rho_s dy'}{2\pi\varepsilon\sqrt{x^2+y^2}}\bar{a_r} \longrightarrow (3)$$

w.k.t. $d\bar{E_x} = \bar{dE}\cos\theta \longrightarrow (4)$

$(x)_{r}^{mo}$

substitute eqn(3) in (4)

$xp \frac{(\frac{\pi}{2}k)+1}{1}$

$$d\bar{E_x} = \frac{\rho_s \cdot dy'}{2\pi\varepsilon(\sqrt{x^2+y^2})}\bar{a_r}\,\frac{x}{\sqrt{x^2+y^2}}$$

$$d\bar{E_x} = \frac{\rho_s \cdot x\, dy'}{2\pi\varepsilon(x^2+y^2)}\bar{a_r}$$

$$\bar{E_x} = \int_{-\infty}^{\infty}\frac{\rho_s \cdot x \cdot dy'}{2\pi\varepsilon(x^2+y^2)}\bar{a_r}$$

$$\bar{E_x} = \frac{\rho_s}{2\pi\varepsilon}\int_{-\infty}^{\infty}\frac{x\,dy'}{(x^2+y^2)}\bar{a_r}$$

$$= \frac{\rho_s}{2\pi\varepsilon}\int_{-\infty}^{\infty}\frac{\cancel{x}\,dy'}{x^{\cancel{2}}(1+(y/x)^2)}\bar{a_r}$$

$$= \frac{\rho_s}{2\pi\varepsilon x}\int_{-\infty}^{\infty}\frac{1}{1+(y/x)^2}dy'\,\bar{a_r}$$

$$= \frac{\rho_s}{2\pi\varepsilon x}\left[\tan^{-1}[+\infty] - \tan^{-1}(-\infty)\right]$$

$$= \frac{\rho_s}{2\pi\varepsilon x}\left[\tan^{-1}\left[y/x\right]_{-\infty}^{\infty}\right]\bar{a_r}$$

$$= \frac{\rho_s}{2\pi\varepsilon x}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right]\bar{a_r}$$

$$= \frac{\rho_s}{2\pi\varepsilon x}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]\bar{a}_r$$

$$= \frac{\rho_s}{2\pi\varepsilon x}\,\pi \cdot \bar{a}_r$$

$$\bar{E}_x = \frac{\rho_s}{2\varepsilon x}\,\bar{a}_r$$

Electric field intensity due to surface charge $\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon}\,\bar{a}_x}$

Note:- In surface charge sheet lies in xy-plane, the field at a point P on z-axis is

$$\bar{E} = \frac{\rho_s}{2\varepsilon}\,\bar{a}_z$$

similarly, xz-plane on y-axis is $\bar{E} = \frac{\rho_s}{2\varepsilon}\,\bar{a}_y$.

* A point charge of 20nc is located at the origin. Determine the magnitude and direction of $\bar{E}$ at the point (1, 3, −4) mts.

at O(0,0,0) (1,3,−4)

sol:

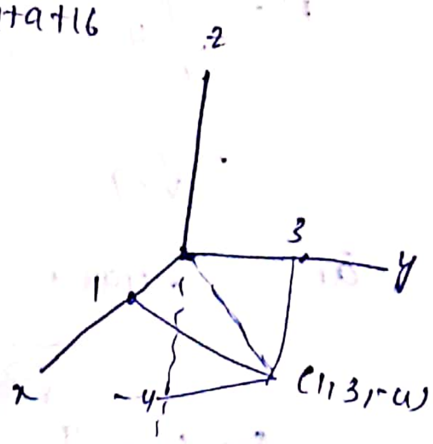
$\bar{R} = (1-0)\bar{a}_x + (3-0)\bar{a}_y + (-4-0)\bar{a}_z$

$\bar{R} = \bar{a}_x + 3\bar{a}_y + -4\bar{a}_z$

$|\bar{R}| = \sqrt{(1)^2+(3)^2+(-4)^2} = \sqrt{1+9+16}$

$= \sqrt{26}$

$q = 20nc = 20\times10^{-9}c$



$$\bar{E} = \frac{Q}{4\pi\varepsilon R^2} \cdot \bar{a_r}$$

$$\bar{a_r} = \frac{\bar{R}}{|\bar{R}|} = \frac{\bar{a_x} + 3\bar{a_y} - 4\bar{a_z}}{\sqrt{26}}$$

$$\bar{E} = \frac{20\times10^{-9}}{4\pi\times8.85\times10^{-12}\times26} \cdot \frac{\bar{a_x}+3\bar{a_y}-4\bar{a_z}}{\sqrt{26}}$$

$$= 1.356\ (\bar{a_x} + 3\bar{a_y} - 4\bar{a_z})$$

$$\bar{E} = 1.356\bar{a_x} + 4.069\bar{a_y} - 5.424\bar{a_z}$$

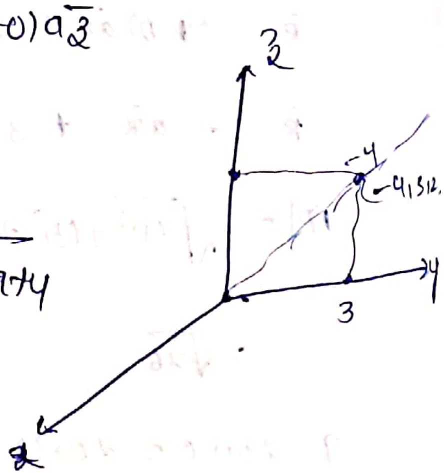
$$|E| = \sqrt{(1.356)^2 + (4.069)^2 + (-5.424)^2}$$

$$= 6.914\ N/C$$

* find E at origin due to a point charge 65 nC is located at point (-4, 3, 2) mt in cartesian co-ordinate system.

sol:-

$$\bar{R} = (-4-0)\bar{a_x} + (3-0)\bar{a_y} + (2-0)\bar{a_z}$$



$$= -4\bar{a_x} + 3\bar{a_y} + 2\bar{a_z}$$

$$|R| = \sqrt{4^2+3^2+2^2} = \sqrt{16+9+4}$$

$$= \sqrt{29}$$

$$\bar{a_r} = \frac{-4\bar{a_x} + 3\bar{a_y} + 2\bar{a_z}}{\sqrt{29}}$$

$$Q = 65\times10^{-9}\ C.$$

$$\bar{E} = \frac{Q}{4\pi\epsilon R^2}\,\bar{a_r}$$

$$= \frac{65\times10^{-9}}{4\pi\,8.85\times10^{-12}\times29} \cdot \frac{-4\bar{a_x} + 3\bar{a_y} - 12\bar{a_z}}{\sqrt{29}}$$

$$= 3.742\,(-4\bar{a_x} + 3\bar{a_y} + 2\bar{a_z})$$

$$= -14.968\,\bar{a_x} + 11.226\,\bar{a_y} + 7.484\,\bar{a_z}$$

$$|\bar{E}| = \sqrt{(-14.968)^2 + (11.226)^2 + (7.484)^2}$$

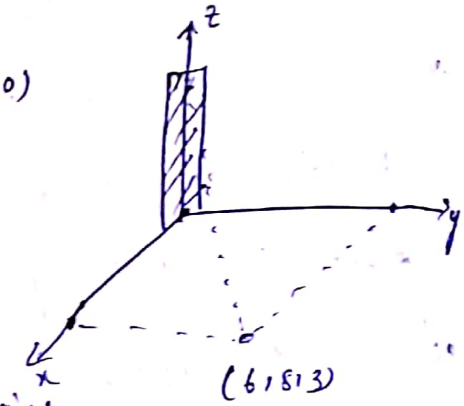
$$= 20.15 \text{ N/C}$$

* A uniform line charge $\rho_L = 20\times10^{-9}$ c/m lies along z-axis. Find the electric field intensity at point P(6,8,3) m.

Sol: Given point P(6,8,3), O(0,0,0)

$$\bar{R} = (6-0)\bar{a_x} + (8-0)\bar{a_y}$$

$$= 6\bar{a_x} + 8\bar{a_y}$$



$$|R| = \sqrt{36+64} \quad \text{(along z-axis)}$$

$$= \sqrt{100} = 10$$

$$\rho_L = 20\times10^{-9}$$

As line charge is along z-axis the electric field intensity cannot have any component along z-direction.

∴

$$\bar{a_r} = \frac{\bar{R}}{|\bar{R}|} = \frac{6\bar{a_x} + 8\bar{a_y}}{10}$$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon \cdot R^2} \cdot \bar{a_r}$$

$$= \frac{20\times10^{-9}}{4\times\pi\times8.85\times10^{-12}\times100} \cdot \frac{6\bar{a_x} + 8\bar{a_y}}{10}$$

$$= 35.967\,(6\bar{a_x} + 8\bar{a_y})$$

$$= 21.58\bar{a_x} + 28.77\bar{a_y}$$

$$|\bar{E}| = 35.964$$

* An infinite sheet in xy-plane from $-\infty$ to $+\infty$ in both directions as a uniform charge density of 10 nano C/m^2. find electric-field intensity at $z = 1$cm.

Soln:- $\rho_s = 10\times10^{-9}$

$$\bar{a_r} = \frac{1}{\sqrt{1}} \bar{a_z}$$

$$\bar{E} = \frac{\rho_s}{2\varepsilon} \bar{a_r} = \frac{10\times10^{-9}}{2\times8.85\times10^{-12}} \cdot \bar{a_z}$$

$$= 0.5649\times10^{3}\,\bar{a_z}$$

$$= 564.9\,\bar{a_z}$$

Note :-

If 2 conducting Spheres are not identical, then the sharing charge when 2 conducting spheres brought into contact and separated by a small distance is given by

$$q_s = \frac{q_1 r_2 + q_2 r_1}{r_1 + r_2}$$

After sharing the charge the force is $f = \frac{q_s \times q_s}{4\pi\epsilon_0 R^2}$

* A small identical conducting spheres have a charge of 2-nanoC and -0.5nC. when they are placed 4cm apart what is the force b/w them? if they are brought into contact and then separated by ~~4~~ cm, what is the force b/w them.

Sol:-

$q_1 = 2 \times 10^{-9} C$

$q_2 = -0.5 \times 10^{-9} C$

$r = 4 cm = 4 \times 10^{-2} mt$

$$q_s = \frac{q_1 r_2 + q_2 r_1}{r_1 + r_2} = \frac{2\times10^{-9} \times 4 \times 10^{-2} + -0.5\times10^{-9} \times 4\times10^{-2}}{4\times10^{-2} + 4\times10^{-2}}$$

$$= \frac{8\times10^{-11} - 2\times10^{-11}}{8\times10^{-2}} = \frac{6\times10^{-11}}{8\times10^{-2}}$$

$= 0.75 \times 10^{-9} C$

$= 0.75 nC$

$$F = \frac{q_s \times q_s}{4\pi\varepsilon R^2}$$

$$= \frac{0.75\times10^{-9}\times0.75\times10^{-9}}{4\times\pi\times8.85\times10^{-12}\times16\times10^{-4}}$$

$$= 3.16\times10^{-6}N$$

$$F = 3.16\,\mu N$$

* A-small identical conducting Spheres have a charge of 3-nanoC and -0.5nC respectively when they are placed 4cm apart. what is the force b/n them if they are brought into contact and then separated by 4cm what is the force

Sol:

$q_1 = 3\times10^{-9}C$, $q_2 = -0.5\times10^{-9}C$

$r = 4\times10^{-2}m$

$$q_s = \frac{q_1r_2+q_2r_1}{r_1+r_2} = \frac{3\times10^{-9}\times4\times10^{-2} + -0.5\times10^{-9}\times4\times10^{-2}}{4\times10^{-2}+4\times10^{-2}}$$

$$= \frac{12\times10^{-11} + -2\times10^{-11}}{8\times10^{-2}}$$

$$= \frac{10}{8}\times10^{-9}$$

$$= 1.25\times nC$$

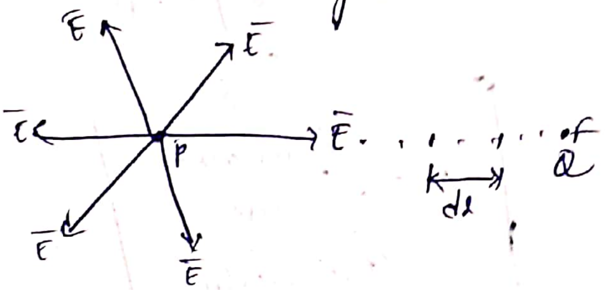
$$F = \frac{Q_S \cdot \times Q_S}{4\pi \varepsilon R^2}$$

$$= \frac{1.25\times10^{-9} \times 1.25\times10^{-9}}{4\times\pi\times 8.85\times10^{-12} \times 16\times10^{-4}}$$

$$= 8.78\ \mu N$$

Work done in a moving charge in electric field:-

- consider an electric field intensity $\bar{E}$ at point 'P'.



Suppose a charge is placed in the electric field intensity ($\bar{E}$) it experience a force. The force due to charge is,

$$F = Q\bar{E} \rightarrow (1)$$

- From the figure to move the charge with a small distance 'dl' to overcome the force. so the work done is defined as

work done = -force x differential length

$= -F \times dl$

$dw = -QE\,dl$

$$\boxed{W = -Q\int_{I.V}^{F.V} \bar{E}\cdot dl}$$

Potential difference :-

- The potential difference is defined as the ratio of work done per unit charge.
- It is indicated by V. are measured in Joule/c.)-V.

Mathematically,

$$V = \frac{W}{Q} = \frac{\text{workdone}}{\text{charge}}$$

$$= \frac{-\cancel{\emptyset}\int \bar{E}\cdot dl}{\cancel{\emptyset}}$$

$$\boxed{V = -\int_{\text{Initialvalue}}^{\text{finalvalue}} E\cdot dl}$$

$$V = -\int_{A}^{B} \bar{E}\cdot dl$$

$$V = -\int_{B}^{(B)} \bar{E}\cdot dl$$

The charge moved from B to A.

Absolute potential:-

let us Assume positive charge to be placed at origin. let R_A is the radial distance of A from origin R_B is the radial distance of B from origin. so the potential difference b/n A and B is,

$$V_{AB} = -\int_{R_B}^{R_A} \bar{E} \cdot d\bar{l}$$

$$= -\int_{R_B}^{R_A} \frac{Q}{4\pi\varepsilon R^2} \bar{a_r} \cdot d\bar{l}$$

$$= \frac{-Q}{4\pi\varepsilon} \int_{R_B}^{R_A} \frac{1}{R^2} d\bar{r} \cdot \bar{a_r}$$

$$= \frac{-Q}{4\pi\varepsilon} \left[\frac{-1}{R} \right]_{R_B}^{R_A} \cdot \bar{a_r}$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon} \left[\frac{1}{R_A} - \frac{1}{R_B} \right] \bar{a_r}$$

$$= \left[\frac{Q}{4\pi\varepsilon R_A} - \frac{Q}{4\pi\varepsilon R_B} \right] \bar{a_r}$$

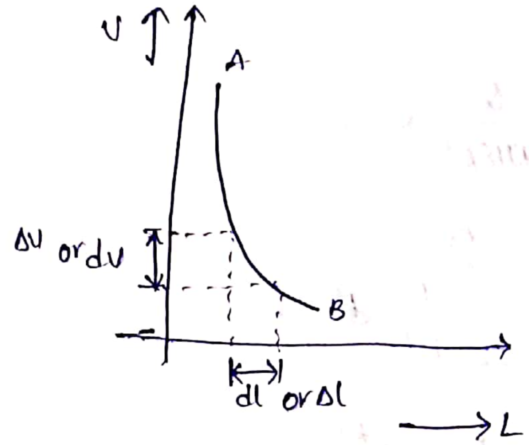
$$V_{AB} = \frac{Q}{4\pi\varepsilon R_A} \bar{a_r} - \frac{Q}{4\pi\varepsilon R_B} \bar{a_r}$$

$$\boxed{V_{AB} = V_A - V_B}$$

where V_A = potential at point A

V_B = potential at point B

* Potential Gradient :-



The graph shows the relation b/n potential V and distance R. Consider a small element AB curve the slope of AB is given as

$$P.G = \frac{dV}{dL}$$

w.k.t

$$\text{Potential } V = -\int \bar{E} \cdot dl$$

$$dV = -\int d\bar{E} \cdot dl$$

$$dV = -\bar{E} \cdot dl$$

$$\frac{dV}{dl} = -E$$

$$P.G = -\bar{E}$$

$$V\nabla = -\bar{E}$$

$$\nabla \cdot V = -\bar{E}$$

$$\boxed{\bar{E} = -\nabla V}$$

* Find the work involved in moving charge of 2C. from (8, 6, -4) to (2, 3, -2) along a straight line in the field of $\bar{E} = x\bar{a_x} + 2y\bar{a_y} - 4z\bar{a_z}$

Sol:- $w = -Q\int E \cdot dl$

$$w = -2\int (x\bar{a_x} + 2y\bar{a_y} - 4z\bar{a_z}) \cdot dx\,dy\,dz$$

$$= -2\int_8^2\int_6^3\int_{-4}^{-2} (x\bar{a_x} + 2y\bar{a_y} - 4z\bar{a_z})\,dx\,dy\,dz.$$

$$= -2\int_8^2\int_6^3 \left[\left[\frac{x^2}{2}\right]_{-4}^{-2} + 2y(x)_{-4}^{-2}\,\bar{a_y} - 4z[x]_{-4}^{-2}\right]dy\,dz$$

$$= -2\int_8^2\int_6^3 \left[\frac{4}{2} - \frac{16}{2}\right] + 2y(-2+4)\,\bar{a_y} - 4z(-2+4)\Big]dy\,dz$$

$$= -2\int_8^3\int_6^3 \left[(2-8) + 4y\,\bar{a_y} - 8z\right]dy\,dz$$

$$= -2\int_8^3 \left[-6[3-6] + \frac{4}{2}[9-36] - 8z[3-6]\right]dz$$

$$= -2\int_8^3 \left[18\right] - 54 + 24z\,]\,dz$$

$$= -2\left[-36[3-8] + \frac{24}{2}[9-64]\right]$$

$$= -2[180 - 660]$$

$$= 960.$$

sol:

$$W = -2\left[\int_{8}^{2} x\,a_x\,dx + \int_{6}^{3} 2y\,dy\,a_y - \int_{-4}^{-2} 4.z\,dz\,a_z\right]$$

$$= -2\left[\left[\frac{x^2}{2}\right]_8^2 a_x + 2\left[y^2\right]_6^3 a_y - 2\left[z^2\right]_{-4}^{-2} a_z\right]$$

$$= -2\left[\left[\frac{4}{4} - \frac{64}{2}\right] a_x + \left[9-36\right] a_y - 2\left[4-16\right] a_z\right]$$

$$= -2\left[(1-32)\,a_x + [-29]\,a_y - 2[-12]\,a_z\right]$$

$$W = -62a_x - 58a_y + 48a_z$$

* Two point charges $q_1 = 2nC$ and $q_2 = 4nC$ are located at $(0,0,-1)$ and $(0,1,0)$ respectively. Determine the potential at point $P(1,1,0)$ due to point charge.

sol:

$$V_1 = \frac{Q}{4\pi\varepsilon R_1}$$

$R_1 = (0,0,-1)$ $\quad R_2 = (0,1,0)$ $\quad P(1,1,0)$

$R_1 = -a_z$

$|r| = 1$

$$V_1 = \frac{2\times10^{-9}}{4\pi\times8.85\times10^{-12}\times1}$$

$= 17.98V$

$R_2 = a_y$

$|R_2| = 1$

$$V_2 = \frac{\cancel{4}\times10^{-9}}{\cancel{4}\times\pi\times8.85\times10^{-12}\times1}$$

$= 35.96\ V$

Total potential $V_{12} = V_1 + V_2$

$= 17.98 + 35.96$

$= 53.94 = 54\ V$

* A point charge of $q = 10\ nC$ is at the origin. Determine the potential difference at $A(1,0,0)$ w.r.t. $B(2,0,0)$

sol:- $V_A = \frac{Q}{4\pi\varepsilon R_1}$

$R_A = a_x$ $\qquad$ $r_B = 2a_x$

$|A_1| = 1$ $\qquad$ $r_B = 2$

$$V_A = \frac{10\times10^{-9}}{4\times\pi\times8.85\times10^{-12}\times1}$$

$= 89.91\ V$

$$V_B = \frac{Q}{4\pi\varepsilon R_A} = \frac{10\times10^{-9}}{4\pi\varepsilon R_B}$$

$$= \frac{10\times10^{-9}}{4\times\pi\times8.85\times10^{-12}\times2} = 44.95$$

$V_{AB} = V_A - V_B = 89.91 - 44.95$

$= 44.96\ V$

* properties of potential difference:-

- potential difference depends only on initial value and final value.
- It doesnot depends on the path between the points.
- It is a scalar quantity V_0
- The units of potential difference is J/C or V
- It is a zero around the closed path.

$$V = -\oint \bar{E}\cdot d\bar{l} = 0$$

* An electric field is given by $\bar{E} = 10y\bar{a_x} + 10x\bar{a_y}$ V/m. Find the potential V. Assume $V=0$ at origin.

sol:-

$$\bar{E} = 10y\bar{a_x} + 10yx\bar{a_y}$$

$$\bar{E} = -\nabla \bar{V}$$

$$\bar{V} = -\int \bar{E}\cdot dl$$

$$dl = dx\bar{a_x} + dy\bar{a_y} + dz\bar{a_z}$$

$$\bar{V} = -\int (10y\bar{a_x} + 10x\bar{a_y})\cdot(dx\bar{a_x} + dy\bar{a_y} + dz\bar{a_z})$$

$$\bar{V} = -\left[\int 10y\,dx + \int 10x\,dy\right]$$

$$= -10xy - 10xy$$

$$\bar{V} = -20xy$$

* A point charge of 4μC is located at free space. find V. If point P is located at P(0.1, 0.2, -0.2).

1. V = 0 at ∞
2. V = 0 at (2, 0, 0)
3. V = 30V at (-0.4, 0, -0)

sol: $Q = 4\mu C = 4\times10^{-6} C$

$P = (0.1, 0.2, -0.2)$

$$V = \frac{Q}{4\pi\varepsilon_0 R} + c \text{ (assumption due to } \infty)$$

$\frac{1}{\infty} \Rightarrow 0$

i) $R = \sqrt{(0.1)^2 + (0.2)^2 + (-0.2)^2}$

Every condition should take from the origin

$\boxed{R = 0.3}$

$$V = \frac{4\times10^{-6}}{4\pi\times8.85\times10^{-12}\times\infty} + c$$

$\Rightarrow c = 0$

$$V = \frac{4\times10^{-6}}{4\times\pi\times8.85\times10^{-12}\times0.3} + 0$$

$= 119890V$

ii) $V = 0$ at $(2, 0, 0)$

$R = \sqrt{2^2} = 2$

$$V = \frac{Q}{4\pi\varepsilon_0 R} + c$$

$$0 = \frac{4\times10^{-6}}{4\times\pi\times8.85\times10^{-12}\times2} + c$$

$c = -17983.6$

$$R = \sqrt{(2-0.1)^2 + (-0.2)^2 + (-0.2)^2}$$

$$= 1.920$$

$$V = \frac{4\times10^{-6}}{4\pi\times8.85\times10^{-12}\times1.920} - 17983.6$$

$$= 749.326V$$

iii) $V = 30$ at $(-0.4, 2, -2)$

$$R = \sqrt{(0.4)^2 + (2)^2 + (2)^2}$$

$$= 2.856$$

$$30 = \frac{4\times10^{-6}}{4\pi\times8.85\times10^{-12}\times2.856} + c$$

$$30 = 12593.56 + c$$

$$c = -12563.56$$

$$R = \sqrt{(-0.4-0.1)^2 + (2-0.2)^2 + (-2-0.2)^2}$$

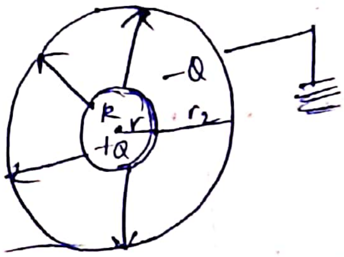
$$= 2.\cancel{85}\cancel{59}\ 88$$

$$V = \frac{4\times10^{-6}}{4\pi\times8.85\times10^{-12}\times2.88} - 12563.56$$

$$= \cancel{56.256}\ V$$

$$= -74.94V$$

* Electric flux (or) displacement flux :-



Dielectric Medium

Two concentric spheres

The Electric flux (or) Displacement flux was first developed by "Michael Faraday", while conducting an experiment on a pair of two concentric spheres as shown in fig.

Let the charge is placed on inner sphere with a distance r and negative charge is placed on outer sphere which may be grounded. so both are isolated from each other.

Finally he concluded that there was a "lines of force from inner sphere to outer sphere. now this displacement is called as flux (or) electric flux.

It is denoted by ψ. units wb.

Electric flux density ($\bar{D}$) :-

It is defined as ratio of electric flux (or) charge per unit surface area. It is denoted by $\bar{D}$.

units :- wb/m^2

Mathematically,

$$\bar{D} = \frac{\psi}{\text{surface area}}$$

$$\boxed{\bar{D} = \frac{\psi}{4\pi R^2}}$$

Relation b/n $\bar{D}$ and $\bar{E}$:-

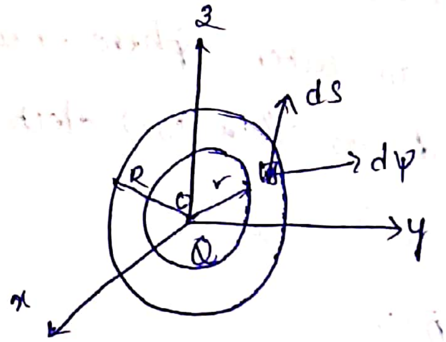
we know that $\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\bar{a_r} \rightarrow (1)$

$$\bar{E} = \frac{\psi}{4\pi\varepsilon_0 R^2}\bar{a_r} \quad [Q = \psi]$$

$$\bar{E}\cdot\varepsilon_0 = \frac{\psi}{4\pi R^2}\bar{a_r}$$

$$\boxed{\bar{E}\cdot\varepsilon_0 = \bar{D}}$$

Gauss law :-



It state that, the surface integral of normal component of electric flux density ($\bar{D}$) over any closed surface is equal to charge enclosed.

$$\boxed{\int_s D\cdot \bar{ds} = Q_{enclosed} = \psi}$$

Proof:- let us consider a point charge of origin an enclosed that charge with a guassian surface (sperical).

→ let us assume a differential surface (ds) and differential flux ('dψ)'

→ we know that electric flux density is

$$\bar{D} = \frac{\psi}{S}$$

$$\bar{D} = \frac{d\psi}{ds}$$

$$d\psi = \bar{D} \cdot ds$$

$$\psi = \int_S \bar{D} \cdot d\bar{s}$$

Area of sphere $4\pi R^2$

$$\psi = D \int_S ds$$

$$\psi = D[s]$$

$$\psi = \frac{Q}{\cancel{4\pi R^2}} [\cancel{4\pi R^2}]$$

$$\boxed{\psi = Q}$$

$$\Rightarrow \int_S \bar{D} \cdot ds = Q_{enclosed} = \psi$$

Maxwell's first eq^n:-

we know that $\rho_v = \frac{Q}{V}$

$$\rho_v = \frac{dQ}{dV}$$

$$dQ = dV \rho_v$$

$$Q = \int_V \rho_v \, dv$$

WKT Gauss law $\int_S \bar{D} \cdot d\bar{s} = Q_{ec.} = \psi$

$$\int_S \bar{D} \cdot d\bar{s} = \int_V \nabla \bar{D} \, dV \longrightarrow (2)$$

apply the divergence theorem for the above eqn

$$\int_{v} \nabla \bar{D} \cdot dv = \int_{v} \rho_v dv$$

substitute in gauss law statement

$$\oint_{s} \nabla \bar{D} \cdot \cancel{ds} = \iint_{v} \rho_v \cdot \cancel{dv}$$

$$\boxed{\nabla \bar{D} = \rho_v}$$

Poisson's Equation:-

we knew that maxwell's first equation is

$$\nabla \bar{D} = \rho_v \longrightarrow (1)$$

wkT the relation b/n $\bar{D}$ and $\bar{E}$ is

$$\bar{D} = \varepsilon \bar{E} \longrightarrow (2)$$

substitute equ(2) in (1)

$$\nabla (\varepsilon \bar{E}) = \rho_v \longrightarrow (3)$$

wkT $\bar{E} = -\nabla V \longrightarrow (4)$

sub eqn (4) in (3)

$$\nabla (\varepsilon (-\nabla V)) = \rho_v$$

$$-\nabla^2 \varepsilon V = \rho_v$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\varepsilon_0}}$$

Replace

note:-

Laplace equation wrt the poissions equation

$$\nabla^2 V = \frac{-\rho_v}{\varepsilon_0}$$

If volume charge density $\rho_v = 0$ then the above equation is

$$\boxed{\nabla^2 V = 0}$$

Application of Gauss law:-

1. Gauss law provides an easy to find electric field intensity $(\bar{E})$ and electric flux density $(\bar{D})$ for symmetrical charges distribution such as

1. point charge
2. line charge
3. surface/sheet charge
4. volume charge

Note: In Rectangular co-ordinate system the Laplace equation is

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

- In polar co-ordinate system the Laplace equation is

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

- In Spherical co-ordinate system the Laplace equation is

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\cdot\frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

→ Determine whether or not the following potential fields satisfy the Laplace equation.

1) $V = r\cos\theta + \phi$ 2) $V = 2x^2 - 3y^2 + z^2$

3) $V = r\cos\phi + z$ 4) $V = r\cos\theta + \phi$

5) $V = 4x^2 - 6y^2 + 2z^2$

Sol:-

1) $V = r\cos\theta + \phi$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\cdot\frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}(r\cos\theta+\phi)\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\cdot\frac{\partial}{\partial \theta}(r\cos\theta+\phi)\right]$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial \phi^2}\left[r\cos\theta+\phi\right]$$

$= 0$

$= \frac{1}{r^2}\frac{\partial}{\partial r}$

ii) $u = 2x^2 - 3y^2 + z^2$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$= \frac{\partial^2}{\partial x^2}\left[2x^2 - 3y^2 + z^2\right] + \frac{\partial^2}{\partial y^2}\left[2x^2 - 3y^2 + z^2\right] + \frac{\partial^2}{\partial z^2}\left[2x^2 - 3y^2 + z^2\right]$$

$$= 4 - 6 + 2$$

$$= 0$$

iii) $u = r\cos\phi + 4z$

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2}$$

$$= \frac{1}{\cancel{r}}\frac{\partial}{\partial r}\left[\cancel{r}\frac{\partial}{\partial r}(r\cos\phi + 4z)\right] + \frac{1}{r^{\cancel{2}}}\frac{\partial^2}{\partial \phi^2}(\cancel{r}\cos\phi + 4z) +$$

$$\frac{\partial^2}{\partial z^2}\left[r\cos\phi + 4z\right]$$

$$= \frac{\partial^2}{\partial r^2}\left[r\cos\phi\right] + \frac{1}{r}\frac{\partial^2}{\partial \phi^2}\left[\cos\phi\right] + \frac{\partial^2}{\partial z^2}(4z)$$

$$= 0 + \frac{1}{r}\left[-\cos\phi\right] + 0$$

$=$

4) $u = r\cos\theta + \phi$

Sol^n:

$$\nabla^2 u = \frac{1}{\cancel{r^2}}\frac{\partial}{\partial r}\left[\cancel{r^2}\frac{\partial}{\partial r}(r\cos\theta + \phi)\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta \times \frac{\partial}{\partial\theta}\left[r\cos\theta + \phi\right]\right] + \frac{1}{r^2\sin^2\theta}\left[\frac{\partial}{\partial x^2}\left[r\cos\theta + \phi\right)\right]$$

$$= \frac{\partial}{\partial r}(\cos\theta) + \frac{1}{r^2\cancel{\sin\theta}}\left[\cancel{\cos\theta}\right.\frac{\partial}{\partial\theta}\left[\cancel{\sin\theta}\left[r \cdot -\sin\theta\right)\right] + \frac{1}{r^2\sin^2\theta}(\cancel{0})\frac{\partial}{\partial\phi}\left[r\cos\theta\right]$$

$$= 0 + \frac{1}{r}\left[-\cos\theta\right] + 0$$

$$= -\frac{\cos\theta}{r}$$

5) $u = 4x^2 - 6y^2 + 2z^2$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$= \frac{\partial^2}{\partial x^2}\left[4x^2 - 6y^2 + 2z^2\right] + \frac{\partial}{\partial y^2}\left[4x^2 - 6y^2 + 2z^2\right] + \frac{\partial}{\partial z^2}\left[4x^2 - 6y^2 + 2z^2\right]$$

$$= \cancel{16}\,8 - 12 + 4$$

$$= 0$$

Potential is $30xyz$ and $\varepsilon = 2.25\varepsilon_0$. Find

i) E at $P(-3,1,2)$ ii) ρ_v at P

Sol: $V = 30xyz$ $\varepsilon = 2.25\varepsilon_0$

$= 30xyz$ $= 2.25 \times 8.85 \times 10^{-12}$

$= 1.99 \times 10^{-11}$

$E = -\nabla V$

$= -\left[\frac{\partial}{\partial x}(30xyz)\bar{a_x} + \frac{\partial}{\partial y}(30xyz)\bar{a_y} + \frac{\partial}{\partial z}(30xyz)\bar{a_z}\right]$

$= -\left[30yz\,\bar{a_x} + 30xz\,\bar{a_y} + 30xy\,\bar{a_z}\right]$

$E(P) = -\left[30(1)(2)\bar{a_x} + 30(-3)(2)\bar{a_y} + 30(-3)(1)\bar{a_z}\right]$

$= -\left[60\bar{a_x} - 180\bar{a_y} - 90\bar{a_z}\right]$

$= -60\bar{a_x} + 180\bar{a_y} + 90\bar{a_z}$

ii) $\rho_v = \nabla \cdot \bar{D}$ $[\bar{D} = \varepsilon\bar{E}]$

$\bar{D} = \varepsilon\left[-60\bar{a_x} + 180\bar{a_y} + 90\bar{a_z}\right)$

$= 1.99 \times 10^{-11}\left[-60\bar{a_x} + 180\bar{a_y} + 90\bar{a_z}\right)$

$\rho_v = \nabla \cdot \left[1.99 \times 10^{-11}\left[-60\bar{a_x} + 180\bar{a_y} + 90\bar{a_z}\right]\right.$

$= 0$

$= 5x^3y^2z$, $\varepsilon = 2.25 \times \varepsilon_0$ find

i) $\bar{E}$ at $P(-3,1,2)$ ii) ρ_v at P

Sol: $V = 5x^3y^2z$ $\varepsilon = 2.25 \times \varepsilon_0$

$= 2.25 \times 8.85 \times 10^{-12}$

$= 1.99 \times 10^{-11}$

$E = -\nabla V$

$$= -\left[\frac{\partial}{\partial x}(5x^3y^2z)\bar{a_x} + \frac{\partial}{\partial y}(5x^3y^2z)\bar{a_y} + \frac{\partial}{\partial z}(5x^3y^2z)\bar{a_z}\right]$$

$$= -\left[15x^2y^2z\,\bar{a_x} + 10x^3yz\,\bar{a_y} + 5x^3y^2\bar{a_z}\right]$$

$$E(P) = -\left[15(-3)^2(1)(2)\bar{a_x} + 10(-3)^3(1)(2) + 5(-3)^3 \times (1)^2 \bar{a_z}\right)$$

$$= -\left[+270\,\bar{a_x} - 540\,\bar{a_y} - 135\bar{a_z}\right)$$

$$= -270\bar{a_x} + 540\bar{a_y} + 135\bar{a_z}$$

ii) $\rho_v = \nabla D$

$$\bar{D} = \varepsilon\left[-270\bar{a_x} + 540\bar{a_y} + 135\bar{a_z}\right]$$

$$= 1.99 \times 10^{-11}\left[-270\bar{a_x} + 540\bar{a_y} + 135\bar{a_z}\right]$$

$$\rho_v = \nabla\left[1.99 \times 10^{-11}\left[-270\bar{a_x} + 540\bar{a_y} + 135\bar{a_z}\right]\right.$$

$= 0$